Homework 7, due 11/12

1. Let $\Omega, \Omega' \subset \mathbf{C}$ be open sets, and $f : \Omega \to \Omega'$ be a holomorphic function. Let $\lambda : \Omega' \to \mathbf{R}$ be a smooth function. Show that

$$\Delta(\lambda \circ f)(z) = |f'(z)|^2 (\Delta\lambda) \circ f(z).$$

2. Let f be a meromorphic function on $\mathbf{C}$ that omits 3 values (one of which may be ∞). Show that f is constant.

3. Let $f, g : \mathbf{C} \to \mathbf{C}$ be holomorphic, such that $f(z)^3 + g(z)^3 = 1$ for all z. Show that f, g are constant.

4. (a) Consider the lattice in $\mathbf{C}$ generated by $\omega_1 = 1, \omega_2 = e^{2\pi i/3}$. Show that in the differential equation

$$(\wp')^2 = 4\wp^3 - g_2\wp - g_3$$

for the corresponding Weierstrass function, we have $g_2 = 0$.

(b) By considering functions of the form $(a + b\wp')/\wp$, show that there exist meromorphic functions f, g on $\mathbf{C}$ such that $f^3 + g^3 = 1$.